

# 6 Measurement

Practical measurement and data collection are important skills within the broader definition of numeracy, and can encourage the development of problem solving skills which are valuable for employment. Practical measurement and data collection in realistic work situations can be interesting, motivating and an enjoyable aspect of numeracy for students.

Data may be collected in the workshop or laboratory, during field work, or through carrying out surveys with groups of people. Measurement may involve the use of measuring instruments including electronic technology.

A range of general numeracy skills may be involved in practical data collection:

- Careful planning should be carried out to ensure that all of the required data is collected.
- It is often necessary to choose appropriate units of measurement, and select suitable equipment for measurement and data recording.
- It may be necessary to choose data from a wider set of possible measurements, and in these cases the selected data needs to be representative of the overall pattern.
- It will be necessary to determine an appropriate level of accuracy for the results which are presented.

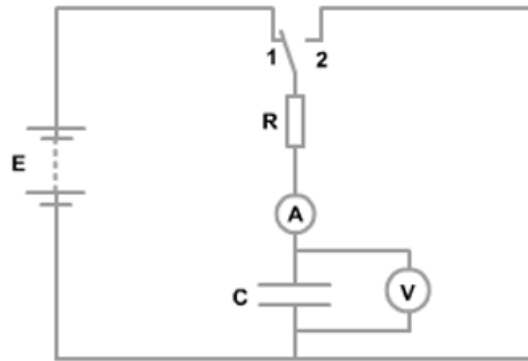
These skills will be important in real work situations when students enter employment.

## Capacitor charging and discharging

Many practical numeracy activities are included in science and technology courses. As an example, we look at experiments carried out by electronics students to investigate the charging and discharging of capacitors, and the application of capacitors in transforming alternating current to direct current.

A capacitor is a component which can store electrical charge. Typically, it consists of two metal plates separated by an insulating material. If a power supply is connected across the capacitor, an electrical charge will build up on the plates, reaching a limiting value which is determined by the size and electrical properties of the capacitor. If the power supply is then removed and the plates linked by an electrical connection, the charge can flow back through the circuit and the capacitor will discharge.

The charging and discharging of a capacitor can be investigated by setting up a circuit consisting of a power supply, capacitor, resistances and switches as shown schematically in figure 82. With the switch in position 1, electrical current flows from the supply, through the resistor  $R$ , and builds up on the plates of the capacitor  $C$ .



**Figure 82:** Circuit for investigation of capacitor charging and discharging

When the switch is moved to position 2, the power supply is disconnected. A circuit is created which allows electrical charge to flow back through the resistor, and the capacitor discharges.

The charging and discharging processes can be investigated by connecting an ammeter in the circuit to measure current, and a voltmeter between the terminals of the capacitor to measure the potential difference across its plates.

When a capacitor (C) is being charged through a resistance (R) to a final potential  $V_0$ , it is found that the voltage (V) across the capacitor at any time  $t$  is given by the equation:

$$\text{capacitor charging (potential difference): } V = V_0 \left[ 1 - e^{-\left(\frac{t}{RC}\right)} \right]$$

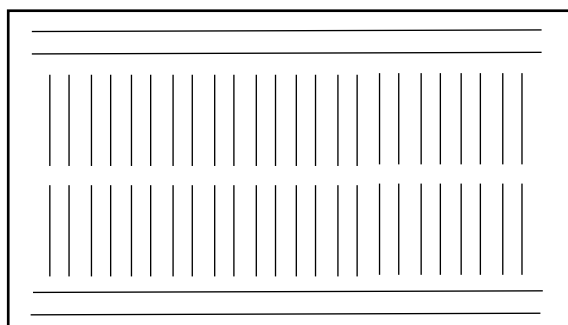
We will examine the mathematics of this equation in more detail in chapter 12 on Calculus. However, we can deduce that for a circuit of low capacitance C and resistance R, the term which gives the power of  $e$ :

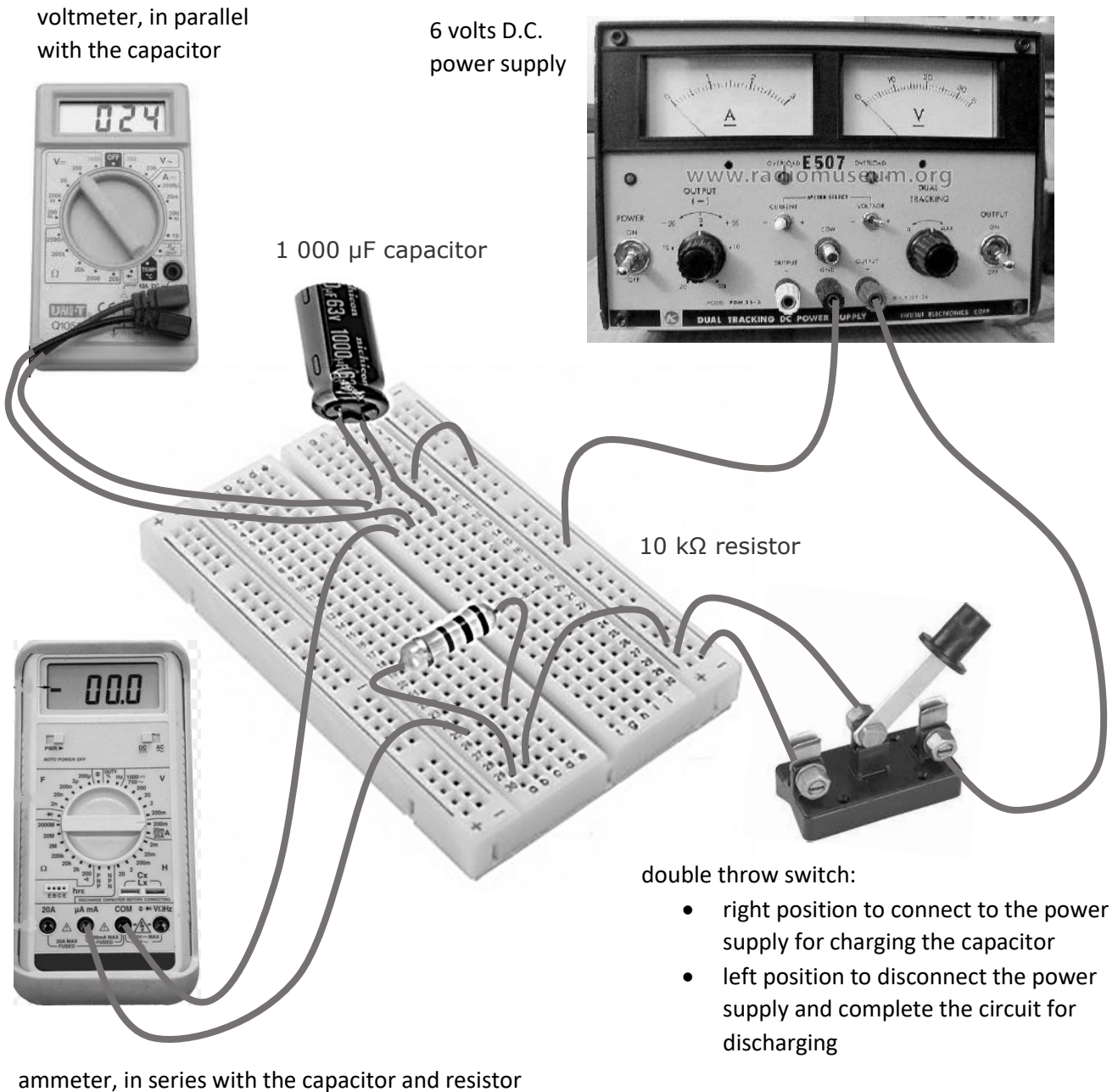
$$-\left(\frac{t}{RC}\right)$$

will have a large magnitude, so the voltage will change rapidly with time  $t$ . This may be too quick to measure. To provide satisfactory experimental results, a large resistance of 10 k $\Omega$  and a large capacitance of 1 000  $\mu\text{F}$  will be chosen for the experiment.

A prototype circuit can be conveniently assembled on an electronics 'breadboard'. This is a block of sockets into which components and wire connections can be easily inserted without the need for soldering. The layout of the breadboard provides fixed electrical connections along the length of the outer set of connector positions, and transverse connections for groups of sockets in the central area of the board.

**Figure 83:** Pattern of electrical connections within the breadboard





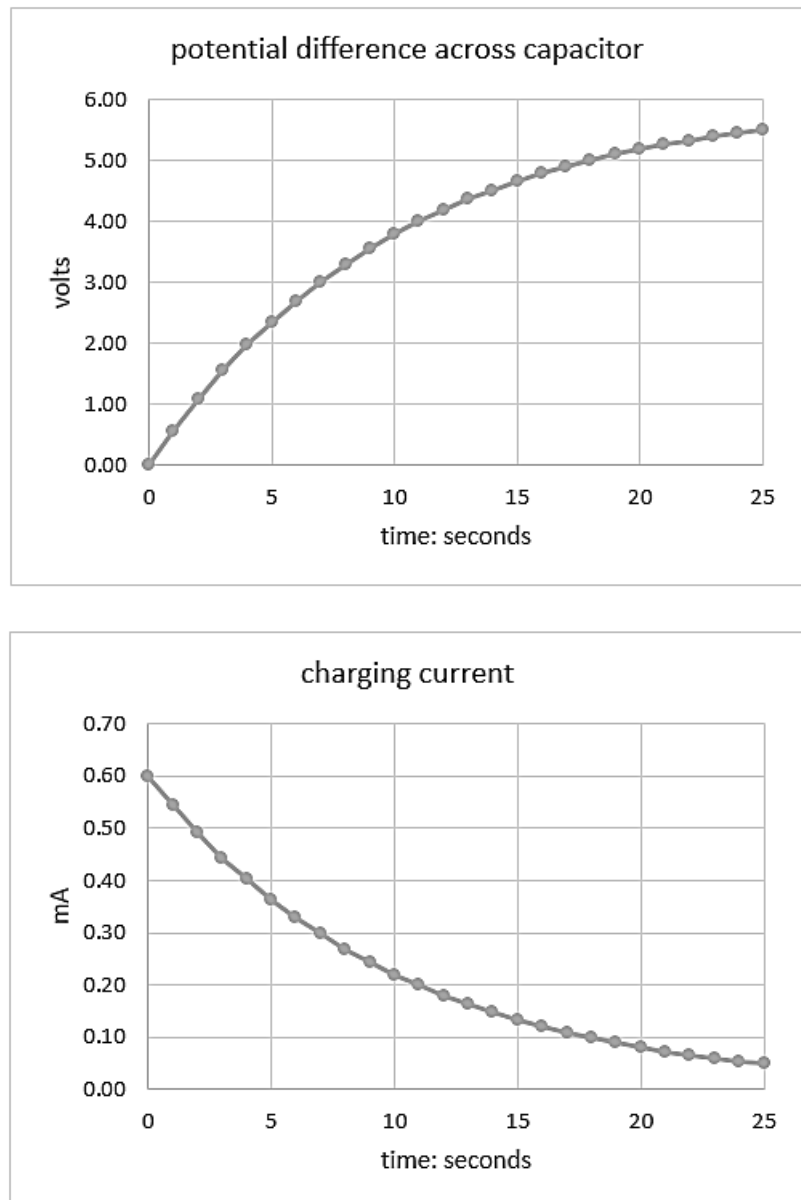
**Figure 84:** Capacitor charging and discharging circuit assembled on a breadboard

With the circuit assembled, the switch can be moved to the charging position and a series of current and voltage readings taken at times recorded with a stop watch. Once the voltage across the capacitor has stabilised, the switch can be moved to the discharging position and a further series of current and voltage readings obtained. Typical results are shown below.

## Capacitor charging

Voltage and current readings are shown in Figure 85 for a period of 25 seconds after closing the switch to complete the charging circuit. It is seen that the voltage builds up across the capacitor plates rapidly in the first few seconds, then at a decreasing rate as the limiting value of the 6 V supply voltage is reached.

t	V	mA
0	0.00	0.60
1	0.57	0.54
2	1.09	0.49
3	1.56	0.44
4	1.98	0.40
5	2.36	0.36
6	2.71	0.33
7	3.02	0.30
8	3.30	0.27
9	3.56	0.24
10	3.79	0.22
11	4.00	0.20
12	4.19	0.18
13	4.36	0.16
14	4.52	0.15
15	4.66	0.13
16	4.79	0.12
17	4.90	0.11
18	5.01	0.10
19	5.10	0.09
20	5.19	0.08
21	5.27	0.07
22	5.34	0.07
23	5.40	0.06
24	5.46	0.05
25	5.51	0.05



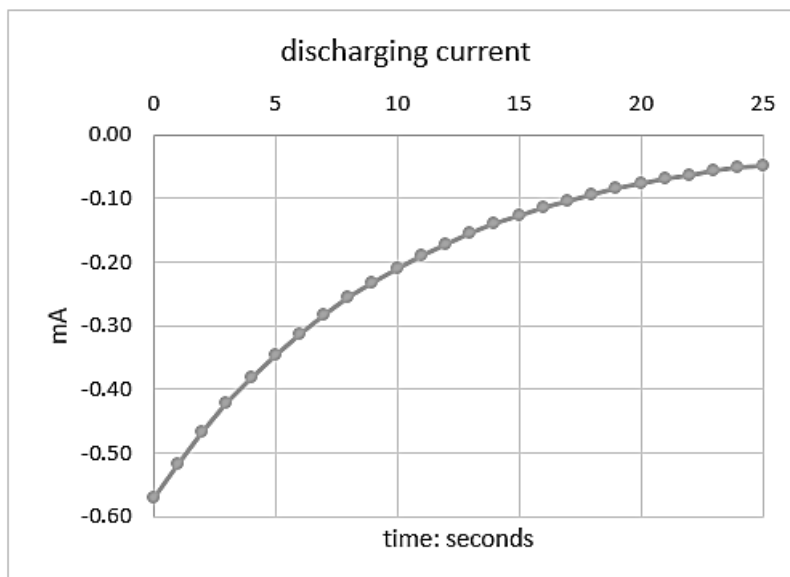
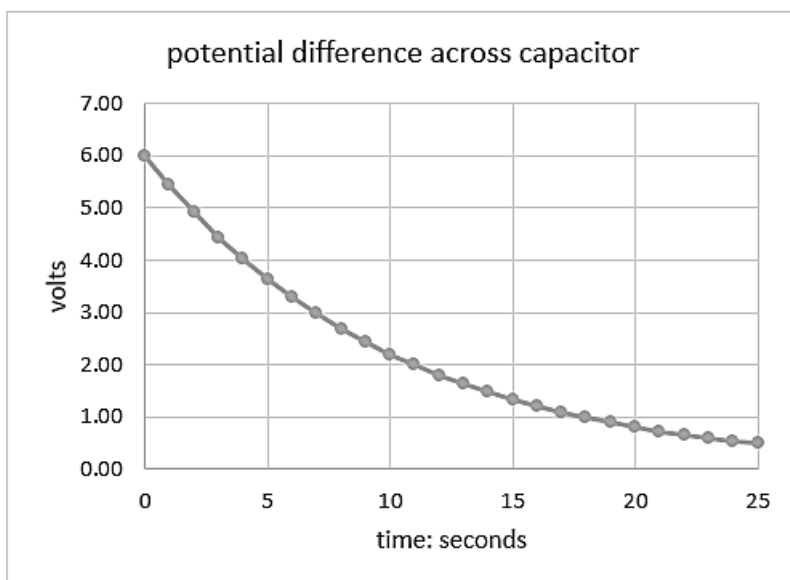
**Figure 85:** Voltage and current readings during capacitor charging

The current flow is initially large, but decreases towards zero as the capacitor becomes fully charged, and can accept no further transfer of electrons through the circuit.

### Capacitor discharging

Voltage and current readings are shown in Figure 86 for a period of 25 seconds after closing the switch to complete the discharging circuit. Voltage across the capacitor plates falls rapidly in the first few seconds, then at a decreasing rate as the charge is lost from the capacitor plates and the potential difference is reduced to zero.

t	V	mA
0	6.00	-0.57
1	5.43	-0.52
2	4.91	-0.47
3	4.44	-0.42
4	4.02	-0.38
5	3.64	-0.35
6	3.29	-0.31
7	2.98	-0.28
8	2.70	-0.26
9	2.44	-0.23
10	2.21	-0.21
11	2.00	-0.19
12	1.81	-0.17
13	1.64	-0.16
14	1.48	-0.14
15	1.34	-0.13
16	1.21	-0.12
17	1.10	-0.10
18	0.99	-0.09
19	0.90	-0.09
20	0.81	-0.08
21	0.73	-0.07
22	0.66	-0.06
23	0.60	-0.06
24	0.54	-0.05
25	0.49	-0.05



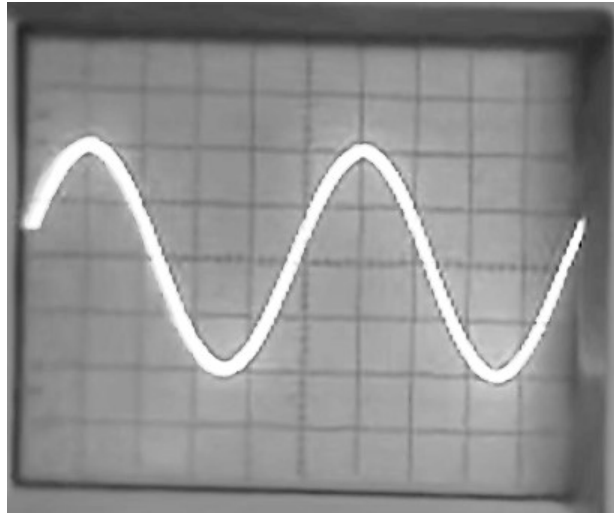
**Figure 86:** Voltage and current readings during capacitor discharging

The discharging current is initially large and in the reverse direction to the charging current. The current decreases towards zero as the capacitor becomes fully discharged and has no further electrons to release through the circuit.

## Transforming Alternating Current to Direct Current

In the next experiment, we will examine the use of a capacitor as part of a circuit to transform alternating current into direct current.

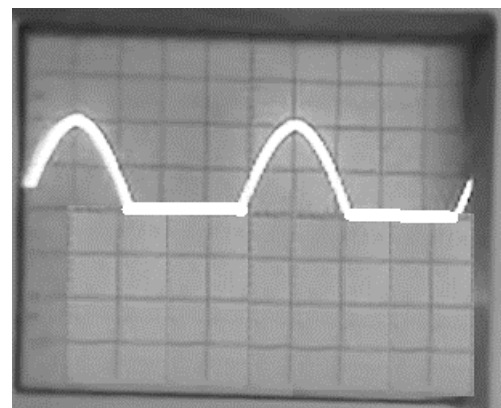
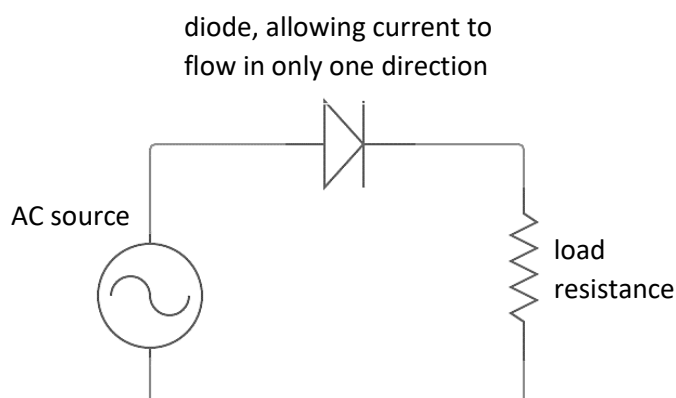
Alternating current has a sine wave form which can be displayed on an oscilloscope screen.



**Figure 87:** Alternating current displayed on an oscilloscope

Controls for the oscilloscope allow the grid to be set to a specified voltage scale on the vertical axis and time scale on the horizontal axis. The oscilloscope has a mechanism to maintain the wave form in a stable position on the screen.

We see from the wave form that each alternating current cycle consists of positive and negative current flow. A first stage in producing direct current is to remove the negative parts of the cycles. This can be achieved by adding a **diode** to the circuit. A diode acts as a valve, allowing current to flow in only one direction.

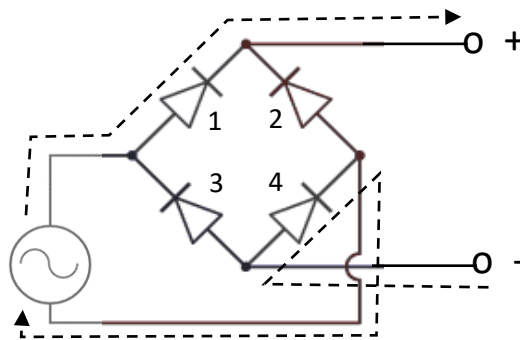


**Figure 88:** Half wave rectification displayed on an oscilloscope

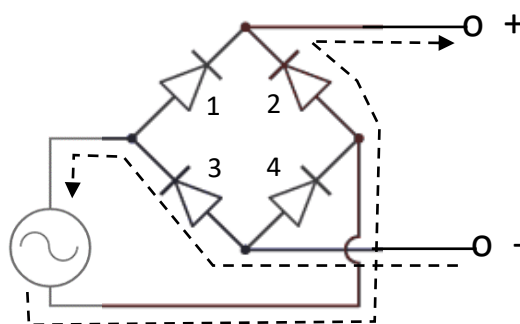
The process in which the negative component of the alternating current is removed is known as **half-wave rectification**. However, the resulting wave form is not sufficiently steady to provide a usable direct current source.

Rather than completely remove the negative parts of the alternating cycles, a more stable output current would be produced if we could simply change the polarity of the negative half-waves to become positive. We would then have a waveform made up of an unbroken sequence of positive peaks. Fortunately, this can be conveniently achieved by means of a linkage of four diodes known as a **bridge rectifier**. Alternating current is applied across two opposite corners of the bridge, with a direct current output connected to the two other corners.

During the positive half-cycle, current flows through diode 1 to the positive direct current terminal, then returns from the negative terminal through diode 4.

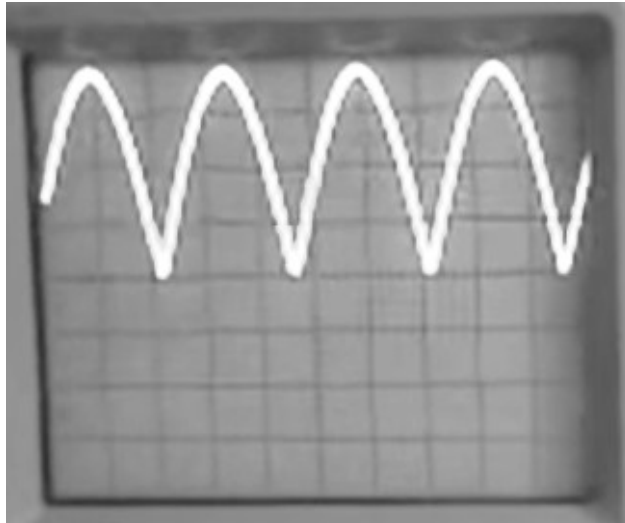


During the negative half-cycle, current flows through diode 2 to the positive direct current terminal, then returns from the negative terminal through diode 3.



**Figure 89:** Current pathways through the bridge rectifier

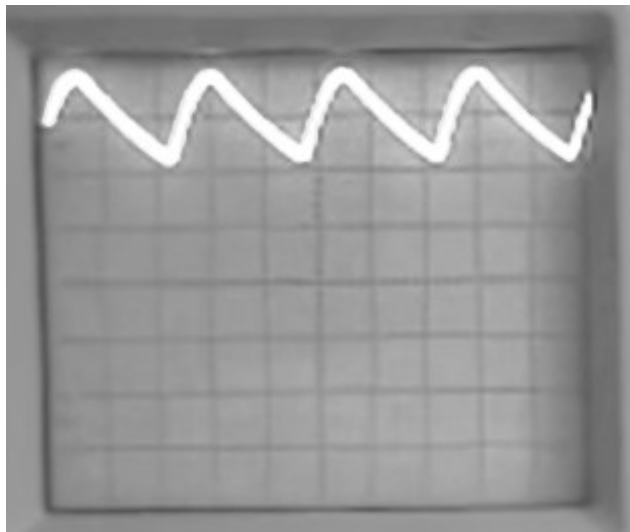
A bridge rectifier can be inserted into the alternating current circuit and the output at the direct current terminals tested by oscilloscope. A wave pattern of repeating positive voltage peaks is produced, as shown in figure 90. This is known as **full-wave rectification**.



**Figure 90:** Full wave rectification

The output is more stable than in the half-wave rectified case, but is still fluctuating between zero and maximum voltage during each half cycle.

To produce an even better direct current output, we add a **capacitor** to the circuit. The objective is to store electrical charge as the A.C. input voltage is rising during each half-cycle, then return this charge to the circuit to maintain a steady output. Figure 91 illustrates the smoothing effect of a relatively small capacitor which can only hold a small amount of electrical charge. A larger capacitor will be able to maintain the direct current output at an almost constant voltage.

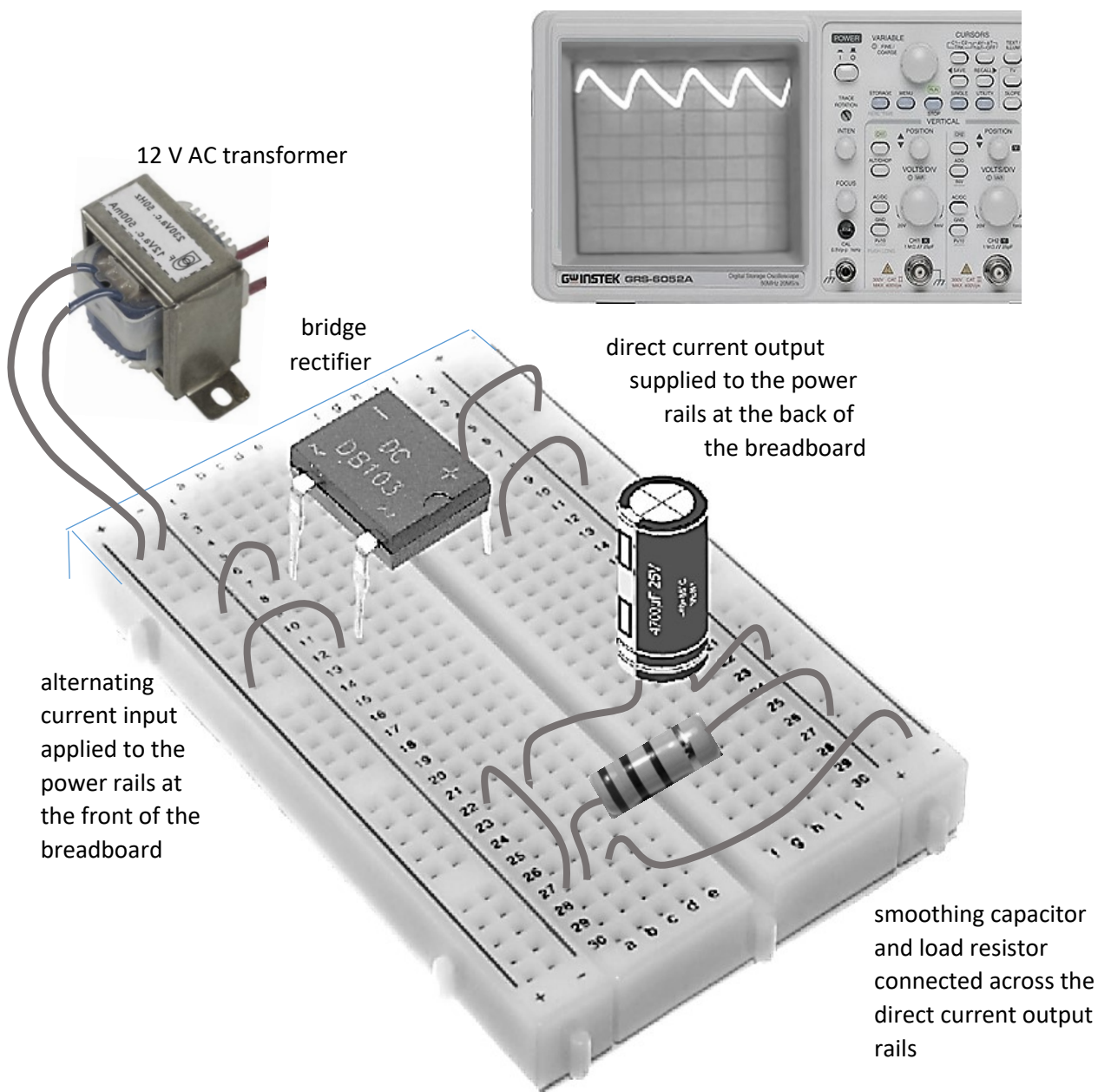


**Figure 91:** Smoothing of the direct current output by means of a capacitor



As in the earlier experiment, it is convenient to assemble the electronic components on a breadboard. Alternating current input of 12V is provided by a transformer. A bridge rectifier is connected across the alternating input, and the full-wave rectified direct current output is supplied to the power rails at the edge of the breadboard. A load resistor and capacitor are connected in parallel across the direct current output lines.

Wave forms at different points in the circuit can be tested by means of an oscilloscope probe.



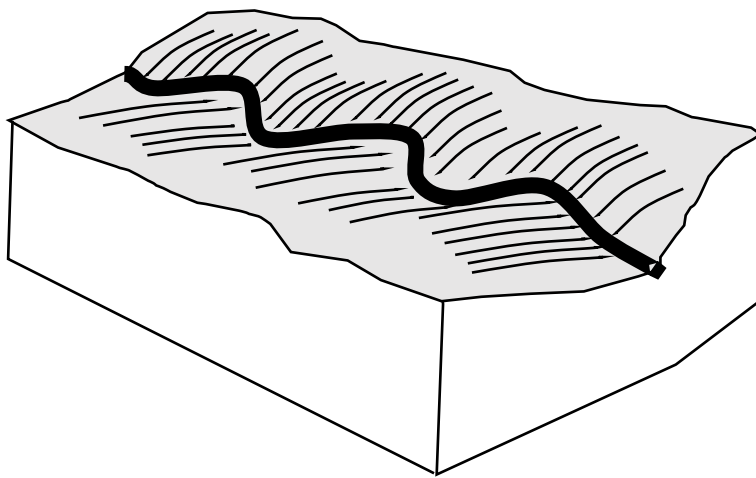
**Figure 92:** Breadboard circuit for investigating direct current output smoothing by capacitor

In Chapter 12 on Calculus, we will look in more detail at the mathematics of current smoothing by capacitor.

## River gravel survey

In the next example, we look at practical data collection by geography students during a study of the distribution of sediment along the course of a river. The area studied is the Mawddach river system, from near its source in the hills of southern Snowdonia, down stream to the tidal limit at the head of the Mawddach estuary.

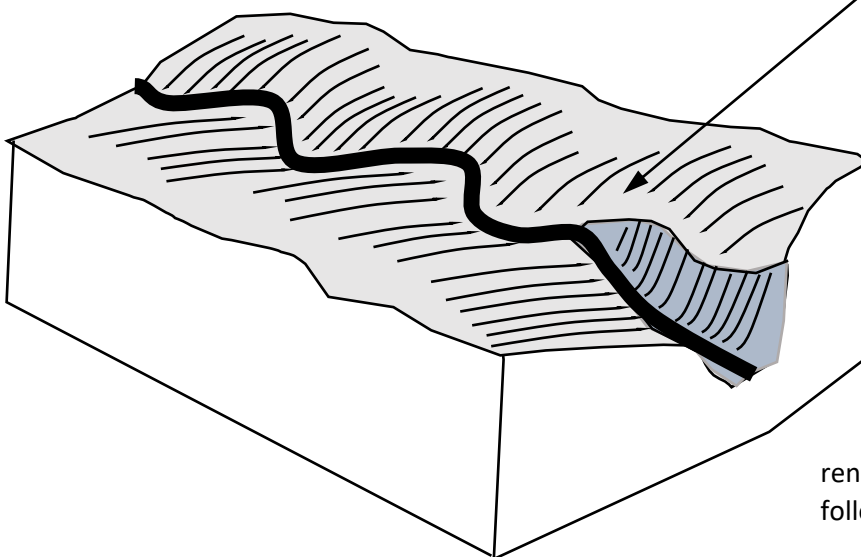
North Wales has had a complex history of uplift and erosion since the late Tertiary geological period. The land surface has undergone several periods of rapid uplift due to earth movements, separated by long periods of stability during which the river system became graded to the current sea level.



development of an early river valley



formation of a knick point  
waterfall: Pistyll Cain

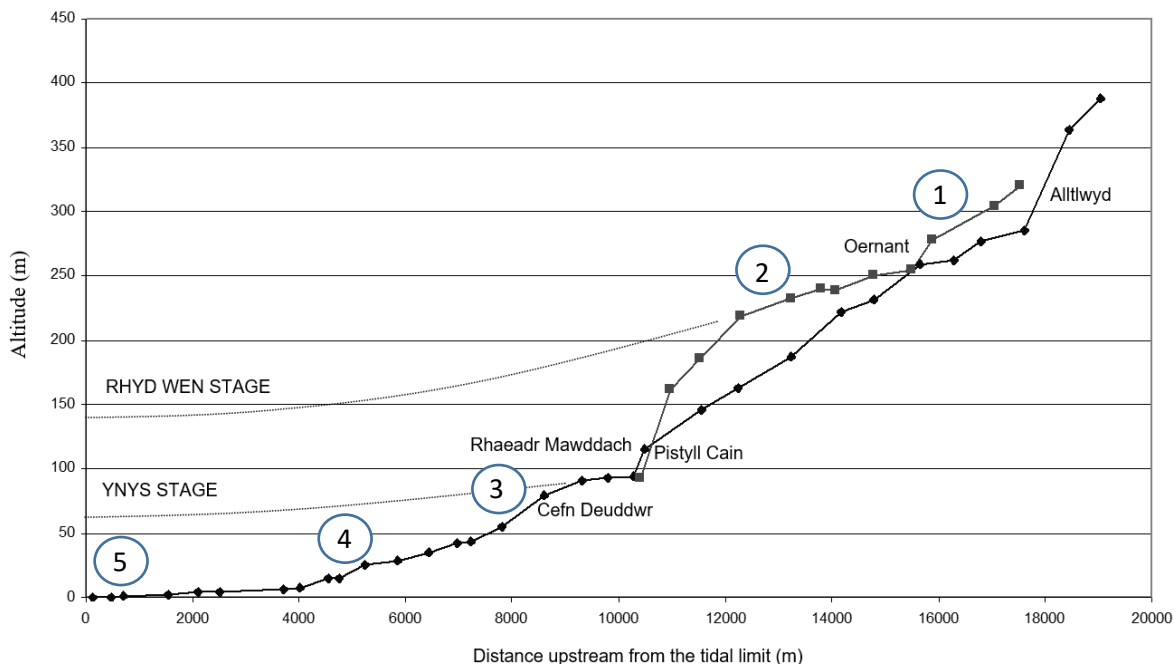


renewed erosion of the lower river valley  
following uplift of the land surface

**Figure 93:** Development of polycyclic relief in the Mawddach river system

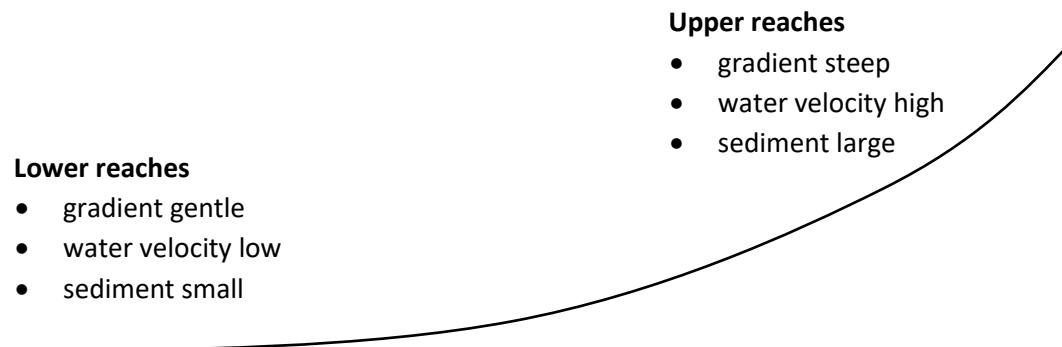
After uplift, renewed erosion begins in the lower course of the river. A **knick point** waterfall may be produced where the focus of maximum erosion is moving upstream.

Figure 94 shows the long profiles of the Afon Mawddach and a major tributary, the Afon Gain. Different sections of the river courses are adjusted respectively to two earlier geological times when the land was lower relative to present day sea level. These are the Rhyd Wen Stage, and the more recent Ynys Stage. During these periods of crustal stability, the river adjusted its base level and nick points moved upstream, producing prominent waterfalls at Rhaeadr Mawddach and Pistyll Cain.



**Figure 94:** Long profiles of the Afon Mawddach and Afon Gain. Reference numbers identify the sediment sampling locations for the fieldwork project.

In an area of simple river development, it might be expected that there will be a gradual transition downstream from a steep, fast flowing stream with a bed of coarse sediment, to a gentle, slower flowing stream with a bed of fine sand or silt.



**Figure 95:** Simple model for river sediment distribution

The objective of the fieldwork described here is to investigate the extent to which the development of polycyclic relief has modified the simple river gradient profile, and how this in turn affects the size distribution of bed sediment along the course of the river.

Five sampling locations were selected on the Afon Gain and along the lower course of the Afon Mawddach, as shown in figure 94:



Locality 1: Oernant, Afon Gain



Locality 2: Bronaber, Afon Gain



Locality 3: Tyddyn Gwladys,  
Afon Mawddach



Locality 4: Ty'n y Groes,  
Afon Mawddach



Locality 5: Llanelltyd, Afon Mawddach

**Figure 96:** Sediment sampling sites on the Afon Gain and Afon Mawddach

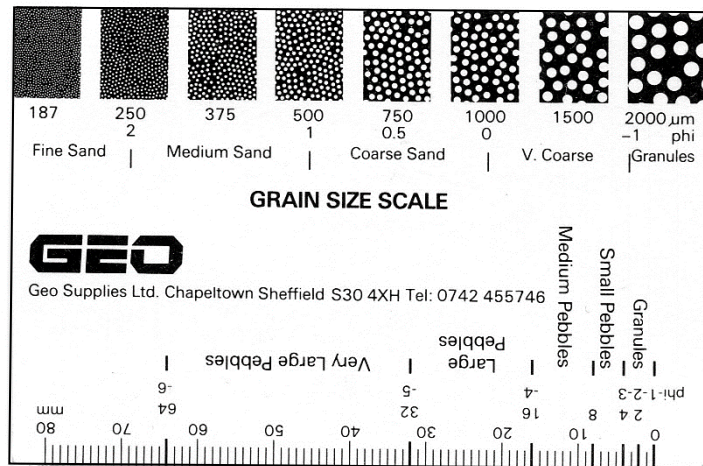
Sediment sizes were recorded at each location. Since sediment sizes can vary widely, from fine silt up to large boulders, it is convenient to use a logarithmic scale of measurement. A system commonly used is the **phi scale**, based on a negative logarithm to the base 2 of the diameter in mm.

$$2^0 = 1, \text{ so phi 0 represent a grain size of 1mm}$$

Grain size is halved for each phi unit:

phi 0	1mm	coarse sand
phi 1	0.5mm	
phi 2	0.25mm	fine sand
phi 3	0.125mm	
phi 4	0.062mm	silt

It is convenient to use a printed waterproof grain size comparator card for determining the phi values for fine sediments, where grain diameters are too small to measure directly.



**Figure 97:** Sediment grain size comparator card

Grain sizes of coarser sediment are represented by negative phi values, increasing in powers of 2:

phi 0	1mm	coarse sand
phi -1	2mm	
phi -2	4mm	
phi -3	8mm	small pebbles
phi -4	16mm	
phi -5	32mm	
phi -6	64mm	very large pebbles
phi -7	128mm	
phi -8	0.25m	
phi -9	0.5m	
phi -10	1m	boulders

If necessary, phi values for boulders larger than 1 metre can be calculated from the base-2 logarithm of the diameter.

Sediment grade was measured at three points across the river channel at each sampling location. To allow for possible poor sorting of the sediment, the maximum, minimum and commonest grain sizes were recorded for each point.

Position	Minimum phi	Commonest phi	Maximum phi
¼ width			
½ width			
¾ width			

Figure 98: Data recording form for sediment grade

River gradient can be conveniently measured by means of a **hydraulic level**. This is simply a long clear plastic tube which is partly filled with water. The tube is extended down the river, with the water level in the upstream end of the tube adjusted to river level. The height **h** by which the water level in the downstream end of the tube rises above river level is recorded, along with the horizontal separation **d** of the ends of the tube.

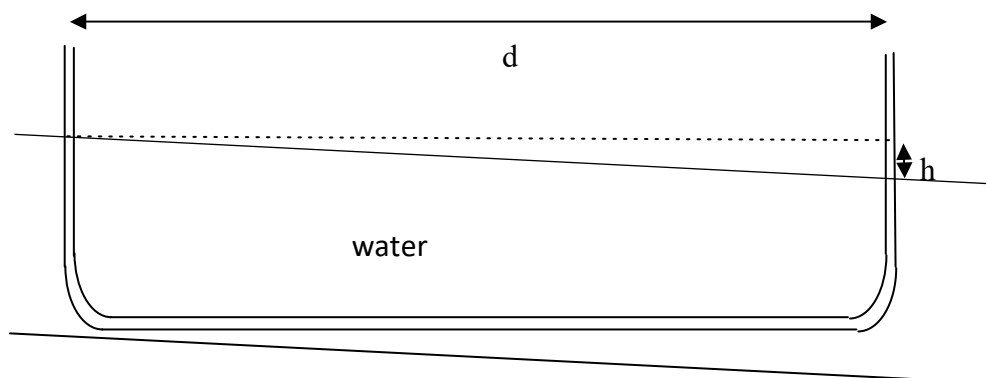


Figure 99: Hydraulic level for determining river gradient

The river gradient angle  $\theta$  can be calculated:

$$\tan \theta = \frac{h}{d}$$

River velocity was measured by a propeller flow meter, calibrated in meters per second. Readings were taken at three positions across the river channel, then averaged.

Position	Velocity m/s
¼ width	
½ width	
¾ width	

Figure 100: Data recording form for river velocity

Readings obtained at the five sampling sites are tabulated in figure 101, and are shown graphically in figure 102.

		Location				
		1	2	3	4	5
d	cm	340	311	323	260	350
h	cm	5	8	3	6	5
<b>Gradient angle</b>	degrees	0.842524	1.473521	0.532144	1.321976	0.818455
<b>Velocity</b>	m/s	0.83	1.05	1.29	1.26	0.9
<b>Sediment Grade</b>						
max	phi	-9.2	-9	-10	-10.5	-9
commonest 1	phi	-7.5	-7	-8	-8	-7
commonest 2	phi	-7.5	-5	-7	-7.5	-5
commonest 3	phi	-7.5	-5	-7	-6.5	-5
min	phi	-1	-2	-1	0	0.5

Figure 101: Results of the river survey

From the graphs, we can make several observations:

Irregular changes occur in the river gradient due to the complex sequence of polycyclic erosion which has occurred in the region. The changes in gradient are also evident from the river long profiles in figure 94 above.

Variations in river velocity follow a more regular pattern. Velocity initially increases downstream from the headwaters to the middle course of the river. This may be related to the increasing volume of water which is carried by the river as additional tributaries join the main stream. Over this middle course, the river is constrained within a narrow valley deeply incised into hard rock.

In the lower course, the flow velocity falls again. We might explain this by observing that the valley floor becomes flatter and the river channel widens. A larger cross sectional area for the river will lead to a lower flow velocity for any particular quantity of water discharge.

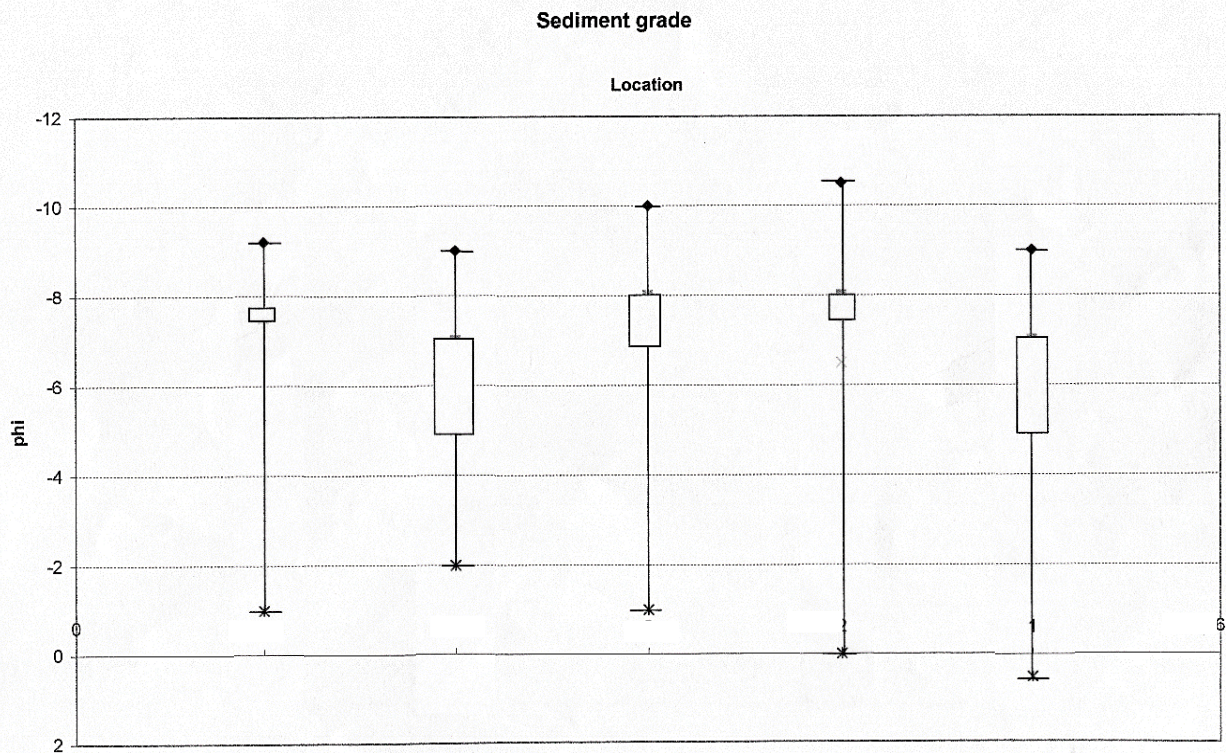
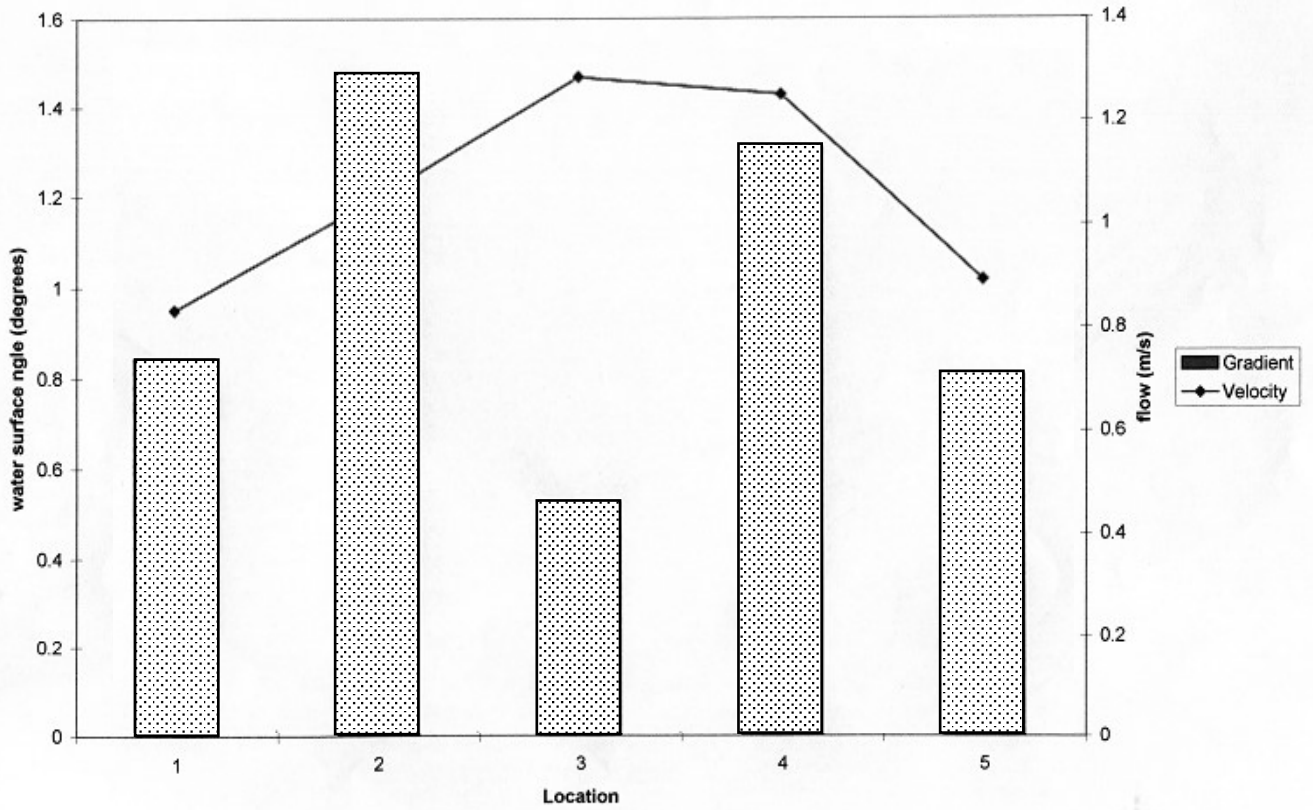


Figure 102: Graphical display of the river survey results



Sediment size is shown as box plots, with the vertical line for each location indicating the range from minimum to maximum phi values. The central box represents the range of commonest grain size values found at the three sampling points across the river channel.

Reasonably good correlation occurs between both the maximum and minimum sediment sizes at each locality and the river flow velocity. There is no apparent correlation with river gradient.

From the results of this fieldwork, it appears that sediment sorting within the river system is primarily dependent on water flow velocity. The survey was carried out during normal river conditions. We would expect the river discharge to be much greater during storm events. However, it is likely that flow velocities will still follow same distribution pattern through the river system, with highest velocities in the middle course where large amounts of flood water are constrained within the narrow rock channels.

A theoretical model can be developed in which sediment moves along a river by three mechanisms:

- **Suspension**, where sediment is carried along with the water flow
- **Traction**, where sediment rolls or slides along in contact with the river bed, due to impact from other moving sediment or pressure from the water flow
- **Saultation**, where sediment moves by a series of jumps as it is picked up by water turbulence then redeposited on the bed.

The effectiveness of the river in moving sediment by all of these mechanisms is closely linked to the water flow velocity:

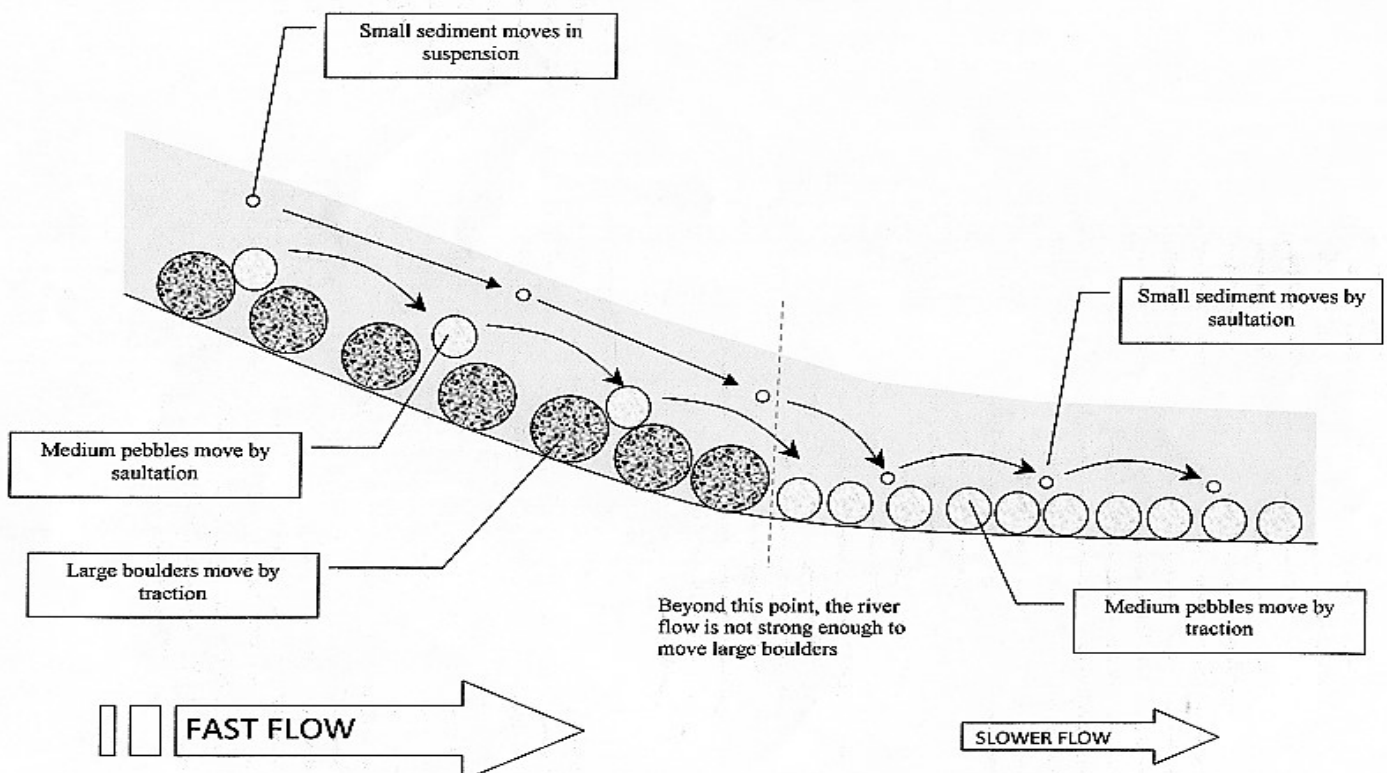


Figure 103: A theoretical model for sediment movement in rivers

## Men's health

For the next example we move away from the experimental measurement of physical quantities, and look instead at the collection of survey data from people. We describe a project carried out by an education student working in the health sector. In preparation for a campaign to promote men's health within the community, a questionnaire survey was carried out to investigate attitudes towards health issues. The series of questions are shown below in figure 104.

A series of questions were devised to determine:

- The current state of the respondent's general health
- Lifestyle of the respondent in relation to healthy living
- Attitudes of the respondent to health issues
- Actions taken by the respondent in the past year to improve their health

A large amount of data can be quickly obtained in a questionnaire survey. Careful thought is needed to devise a strategy for converting this raw data into information useful for decision making.

When completed questionnaires were received, responses were allocated scores:

For most questions, a positive score could be given to responses indicating a good health outcome, and a negative score given to a bad health outcome. A neutral response was scored as zero.

In some cases, the significance of one question depends on the response to a previous question. For example, if the respondent is currently overweight, then it would be a positive outcome to be losing weight, but negative to be gaining weight. The reverse would be true for a respondent who was currently underweight.

In some cases, a question score is calculated as the total of all the positive health factors selected by the respondent from a list of multiple options.

The survey was carried out in various towns and villages in mid-Wales. Data was collected from a total of 34 respondents covering a wide variety of occupations and ages.

Results were tabulated in a spreadsheet. Summary scores were then calculated for each respondent to show:

- current overall state of health
- extent of healthy living
- attitude towards health issues
- actions taken for health improvement

## HEALTH QUESTIONNAIRE

Question		Score
A	How would you describe your current state of health? <ul style="list-style-type: none"> <li>• Very good</li> <li>• Fairly good</li> <li>• OK</li> <li>• Poor</li> </ul>	2 1 0 -1
B	Compared to the previous 12 months, do you feel that your health has improved? <ul style="list-style-type: none"> <li>• Improved</li> <li>• Stayed the same</li> <li>• Worsened</li> </ul>	1 0 -1
C	Are you interested in trying to improve your health? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Unsure</li> </ul>	1 -1 0
D	What would you like to do during the next 12 months to improve your health? <ul style="list-style-type: none"> <li>• Increase exercise</li> <li>• Improve diet</li> <li>• Lose weight</li> <li>• Reduce/give up smoking</li> <li>• Reduce/give up drinking</li> <li>• None apply</li> </ul>	<total> 1 1 1 1 1 0
E	If available, which of the following would you use? <ul style="list-style-type: none"> <li>• Physical check-up/health MOT</li> <li>• Advice/information about health by trained health professionals</li> <li>• None apply</li> </ul>	<total> 1 1 0
F	How do you rate your present physical condition? <ul style="list-style-type: none"> <li>• Very good</li> <li>• Good</li> <li>• OK</li> <li>• Poor</li> </ul>	2 1 0 -1
G	Do you take enough exercise to remain healthy? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Unsure</li> </ul>	1 -1 0
H	Compared to the previous 12 months, has your exercise level <ul style="list-style-type: none"> <li>• Increased</li> <li>• Stayed the same</li> <li>• Decreased</li> </ul>	1 0 -1
I	Which of the following describes you best? <ul style="list-style-type: none"> <li>• Underweight</li> <li>• OK</li> <li>• Overweight</li> </ul>	-1 0 -1

J	Compared to the previous 12 months, has your weight altered? <ul style="list-style-type: none"> <li>• Increased</li> <li>• Stayed the same</li> <li>• Decreased</li> </ul>	<better> 1 <worse> -1 <same> 0
K	Would you like to change your weight? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Unsure</li> </ul>	<better> 1 <same> 0
L	Do you feel that you have a healthy diet? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Don't know</li> </ul>	1 -1 0
M	Compared to the previous 12 months, is your diet more healthy? <ul style="list-style-type: none"> <li>• More healthy</li> <li>• Less healthy</li> <li>• Same</li> </ul>	1 -1 0
N	Would you like to improve your diet or learn more about healthy eating? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Unsure</li> </ul>	1 0 0
O	Do you smoke? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> </ul>	-3 0
P	Would you like to give up smoking? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Not applicable</li> </ul>	1 0 0
Q	How often do you drink alcohol during a week? <ul style="list-style-type: none"> <li>• Nil</li> <li>• 1-2 days</li> <li>• 3-4 days</li> <li>• 5-7 days</li> </ul>	0 -1 -2 -3
R	Would you like to reduce your alcohol intake? <ul style="list-style-type: none"> <li>• Yes</li> <li>• No</li> <li>• Not applicable</li> </ul>	1 0 0
S	What is your age?	<number>

**Figure 104:** Men's health survey questionnaire, showing score values for responses

Results of the survey show wide variations between different respondents, which in itself is a significant outcome. Health professionals will need to be very flexible in their approaches if they are to be successful in supporting the whole community.

General health within the working population does not seem to be linked to age. A majority of respondents of all ages consider their current health to be satisfactory, rather than particularly good or particularly poor.

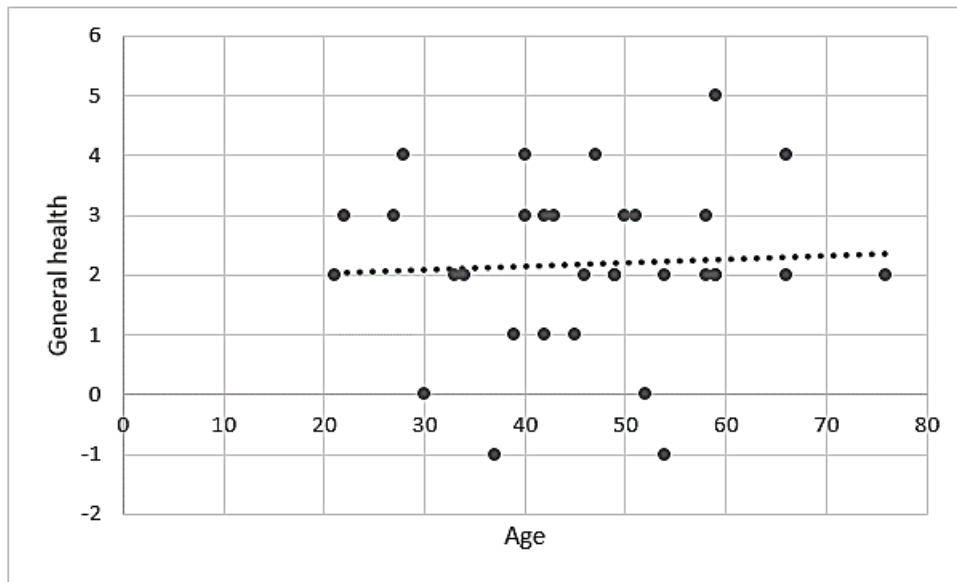


Figure 105: Comparison of general health against age

A small, but perhaps important, positive correlation seems to be present between general health and a healthy life style. This result can reassure health professionals that there are benefits to be achieved from encouraging improvements in lifestyle.

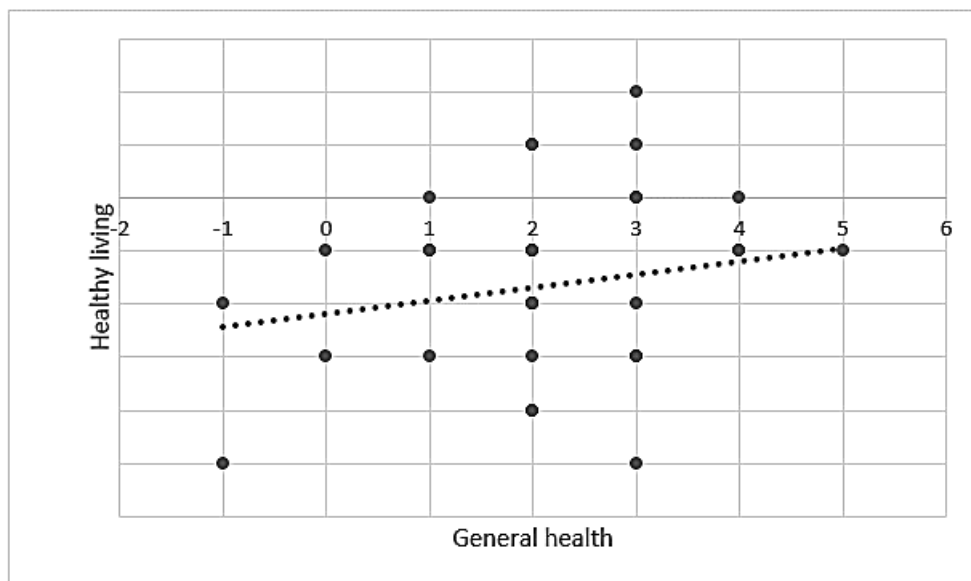
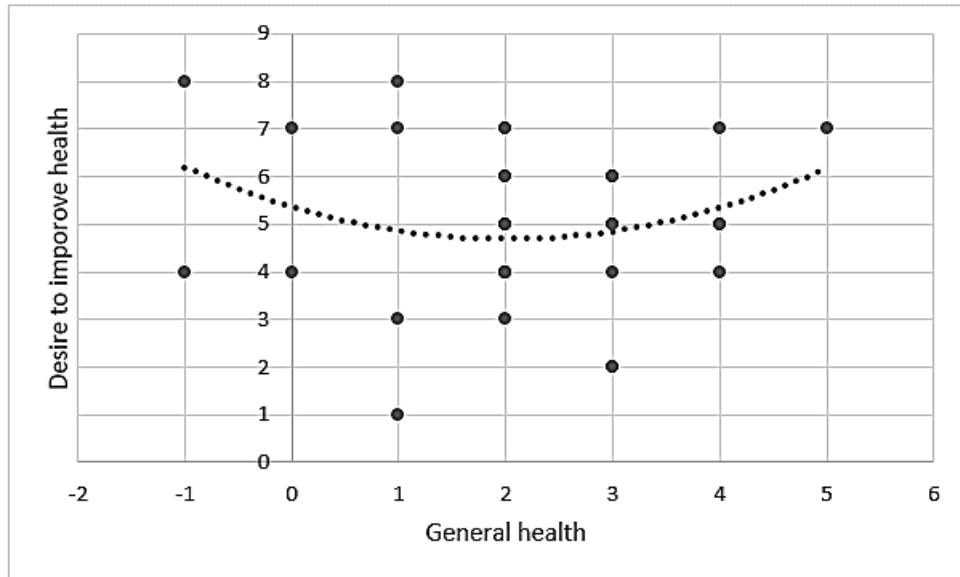
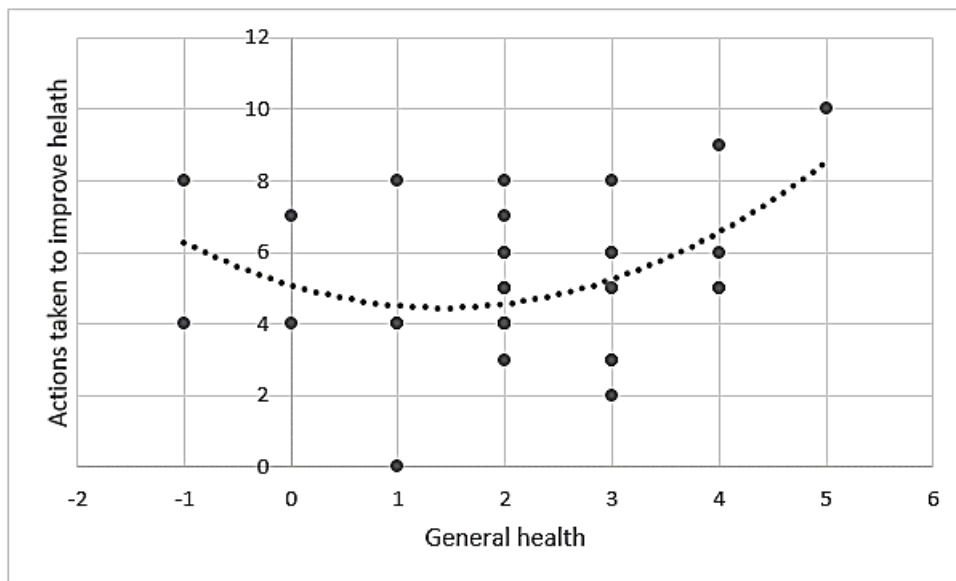


Figure 106: Comparison of healthy living against general health

Interesting results are obtained when comparing respondents' general health with the desire to improve their health, and the positive health actions taken in the last year. Whilst showing wide variations, the best fit lines in both cases appear to be polynomial curves (figures 107 and 108).



**Figure 107:** Comparison of desire to improve health against general health



**Figure 108:** Comparison of actions taken to improve health against general health

The implication of these results is that the men most wishing to improve their health and likely to take action to do this are either:

- those with poorer health, who may be worried that action is needed
- those with very good health, who already have an enthusiasm for health issues

Perhaps the conclusion for health professionals is that there will be most difficulty in motivating the central group who have moderately good health. This group may have no particular health worries, nor any great enthusiasm to develop a healthier lifestyle.

## Building a wind turbine

Construction projects can provide excellent opportunities for developing a range of numeracy skills. These may include problem solving and communication of mathematical ideas, in addition to developing the complex and specialist mathematics required in particular vocational areas.

Our next example illustrates the construction of a small wind turbine by a group of engineering students. This is based on a design developed at the Centre for Alternative Technology, Machynlleth. The turbine has a timber blade construction. The electrical alternator and other metal components were fabricated by the students in the engineering workshop.



**Figure 109:** The completed wind turbine prototype

Before beginning construction, a design for the turbine was created using Solid Works computer aided design software, as shown in figure 110. This allowed measurements to be transferred from scale drawings and checked in the solid model. Students were able to consider the construction techniques which would be needed for the metal and timber parts, and the way in which the electrical alternator would be assembled. Agreement was reached between members of the team on individual tasks and responsibilities when carrying out the work.

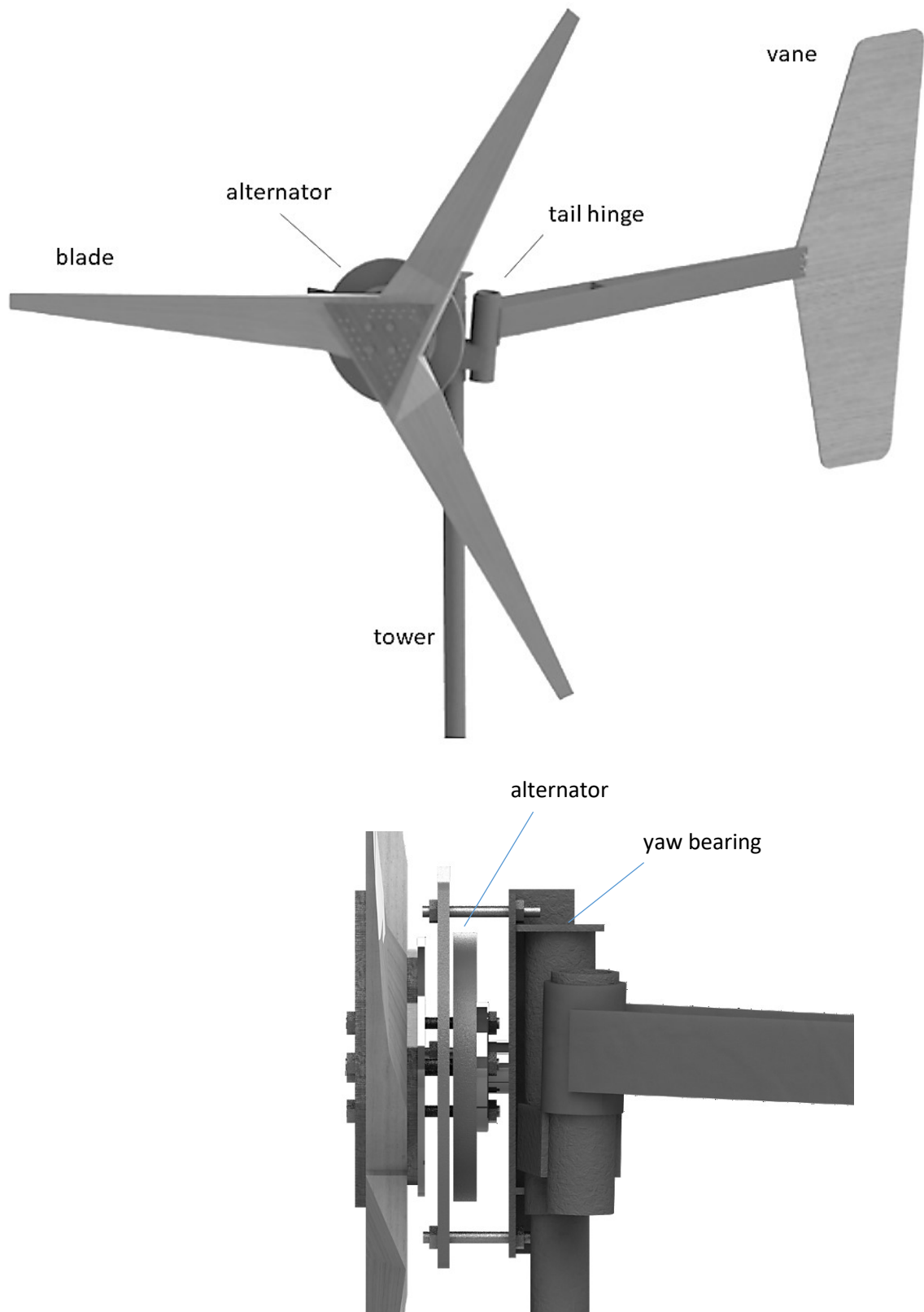
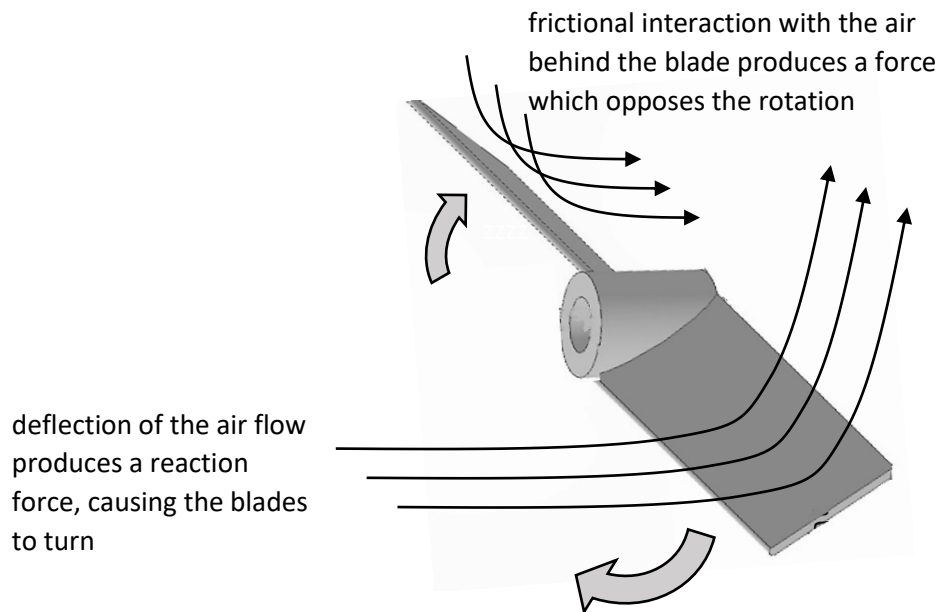


Figure 110: Design for the wind turbine



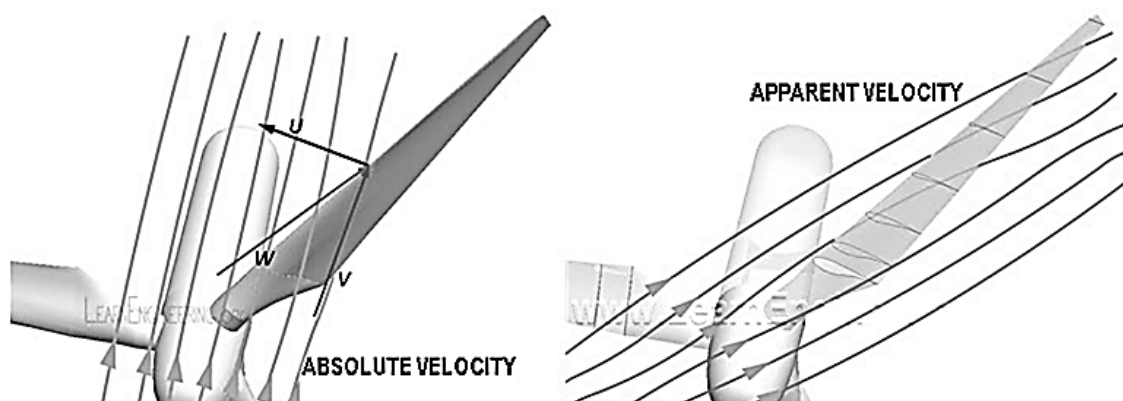
The first objective of the project was to produce a working prototype of the wind turbine according to the plans provided. However, the student group also had the opportunity to investigate the theoretical background to turbine design and consider ways of improving the efficiency and output from the machine.

The simplest design of wind turbine would have flat blades inclined to the wind approach direction. However, this design has a low efficiency due to frictional forces behind the blades which oppose the motion. Flat blade turbines are only suitable for use at slow wind speeds.



**Figure 111:** Forces operating for a flat blade turbine

Most practical wind turbines now use blades which have an aerofoil cross section, similar to an aircraft wing. To understand how these blades extract energy from an airflow, we need to consider the situation when the turbine blade is rotating.

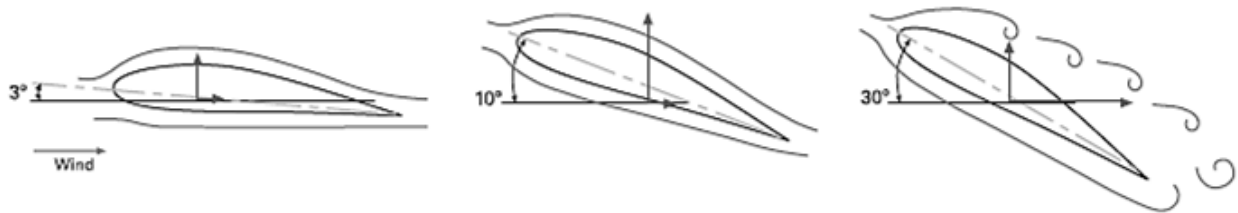


[www.learnengineering.org/2013/08/Wind-Turbine-Design.html](http://www.learnengineering.org/2013/08/Wind-Turbine-Design.html)

**Figure 112:** Air flow past a rotating turbine blade

Although air is approaching the front of the wind turbine tower, the turbine blades are turning into the airflow. If an observer was able to travel around with the turbine blade, they would experience the airflow as approaching at an angle from ahead of the blade, then passing-by behind the blade as it rotates. The difference between the absolute direction of air flow and the apparent direction will increase with the speed of rotation of the blades.

Wind turbines make use of the apparent air flow over the aerofoil shaped blades to produce lift forces in a similar way to an aircraft wing. The lift force provides the torque to rotate the turbine blades.



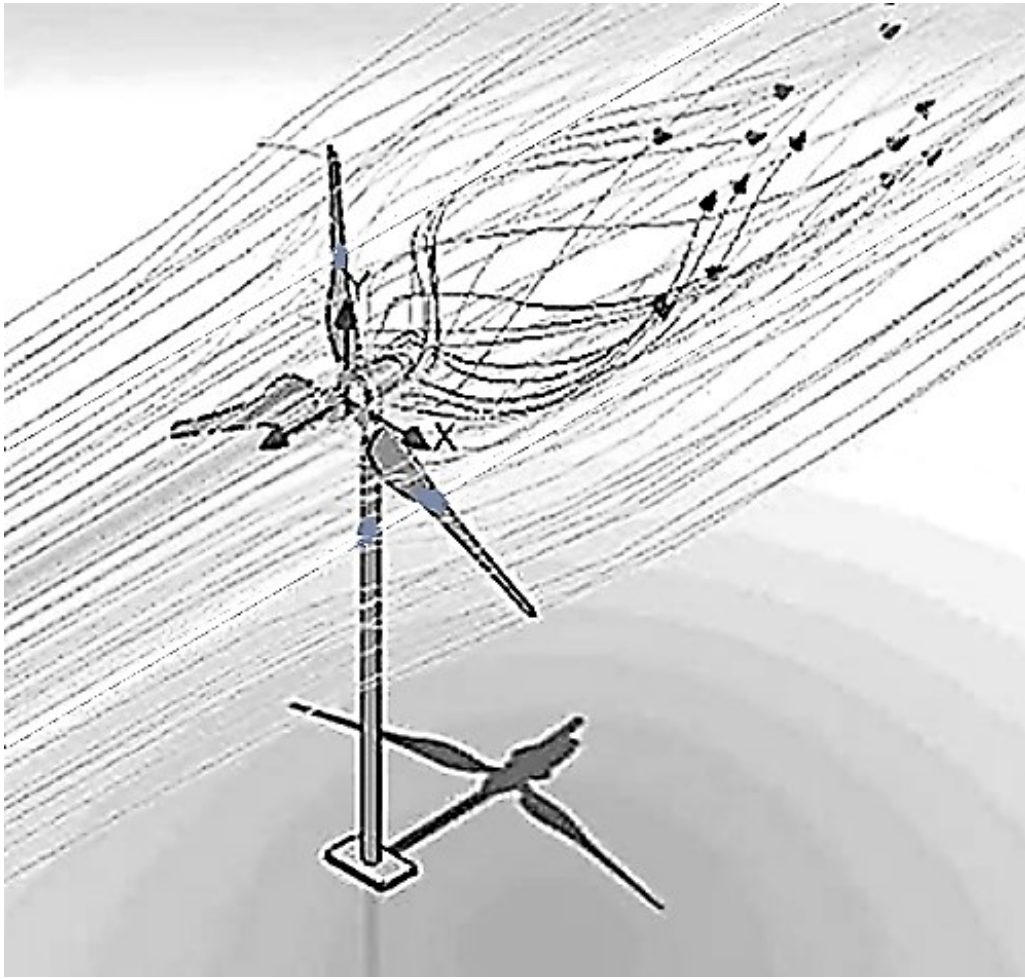
[www.gurit.com/files/documents/2aerodynamicspdf.pdf](http://www.gurit.com/files/documents/2aerodynamicspdf.pdf)

**Figure 113:** Angle of attack for an aerofoil turbine blade

The angle of the aerofoil in relation to the apparent air flow, known as the **angle of attack**, determines the amount of lift force produced. The lift force is small for a low angle of attack, increasing to a maximum when the angle is about  $10^\circ$ . If the angle of attack is increased beyond this, the aerofoil begins to produce turbulence. Energy is lost and the lift force is reduced.

Advanced designs of turbine provide a mechanism by which the angle of attack of the blades can be automatically adjusted to the current wind speed, so that the lift forces are maximised. For turbines with non-adjustable blades, the angle of attack should be calculated to suit the optimum wind speeds for the turbine site. A further complexity occurs because the outer sections of a turbine blade furthest from the rotation axis are travelling faster than the sections near the axis. This difference can be accommodated by introducing a twist in the aerofoil section along the blade. The angle of attack is increased towards the tip, where blade movement is fastest and the difference between the absolute and apparent air flow directions is greatest.

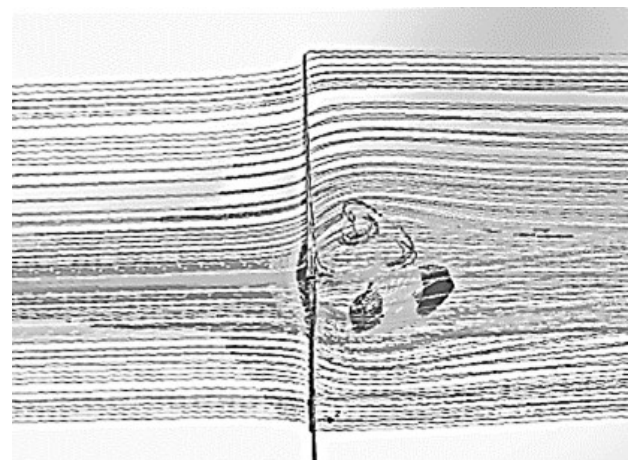
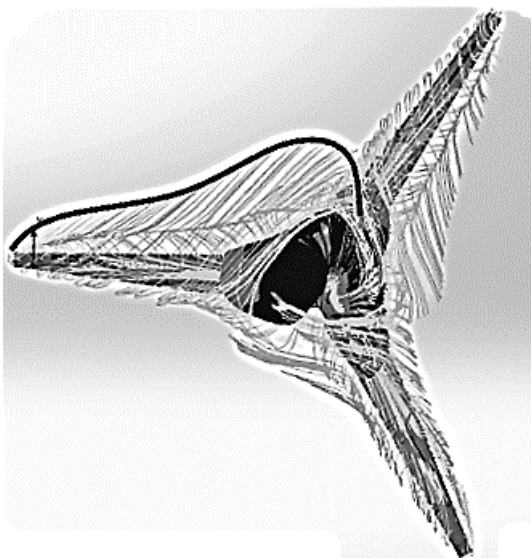
Another design consideration is the speed of rotation of the blades for particular wind speeds. The engineer may choose to allow the turbine to rotate very freely, generating little torque, or to rotate slowly producing high torque. The speed of the blade tips relative to the free wind speed before the wind is slowed down by the turbine is called the **tip speed ratio**. The choice of tip speed ratio can affect the overall power output from the wind turbine. Low blade speeds can produce large amounts of rotational turbulence behind the turbine, reducing the amount of energy available (figure 114).



[www.youtube.com/watch?v=mrSci7wG27o](http://www.youtube.com/watch?v=mrSci7wG27o)

**Figure 114:** Rotational turbulence produced behind a wind turbine

The wind turbine will also cause a reduction in wind speed as air is deflected by the structure and the swept area of the rotating blades. Further approaching air will be deflected around the slower air mass. The students were able to investigate these effects in their Solid works model, and make modifications to the design to reduce turbulence:



**Figure 115:** Air flow model for the wind turbine prototype

Theoretical considerations have led to a mathematical relationship (National Instruments, 2014) for the amount of power which can be extracted from a wind turbine. A factor termed the power coefficient  $C_p$  is obtained from a chart linking turbine blade tip ratio and angle of attack  $\theta$

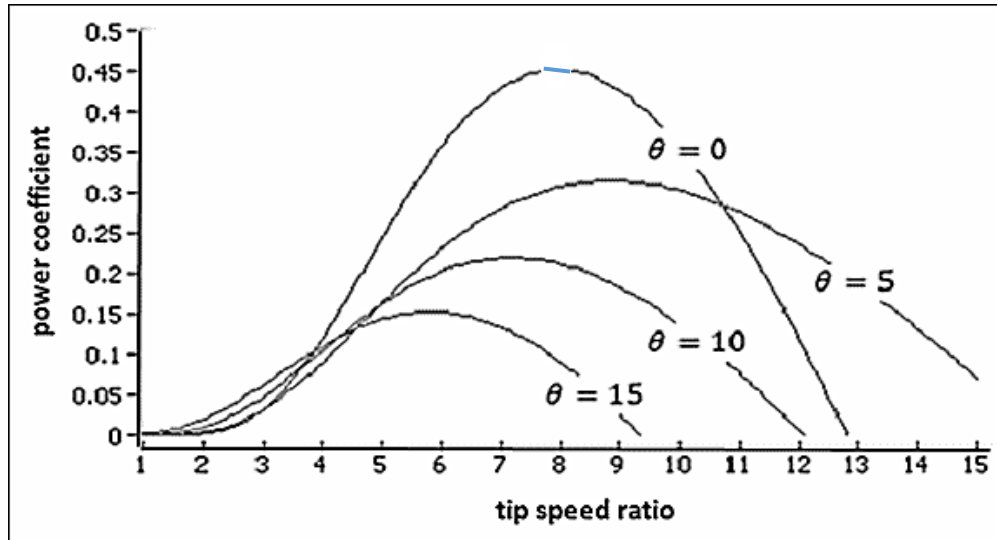


Figure 116: Chart for determination of power coefficient

Once a power coefficient has been estimated, the power produced by the turbine at particular wind speeds can be calculated from the formula:

$$P_{mech} = 0.5 \rho \pi R^2 v^3 C_p$$

where:

$P_{mech}$  is the mechanical power

$\rho$  is the air density

$R$  is the turbine's radius

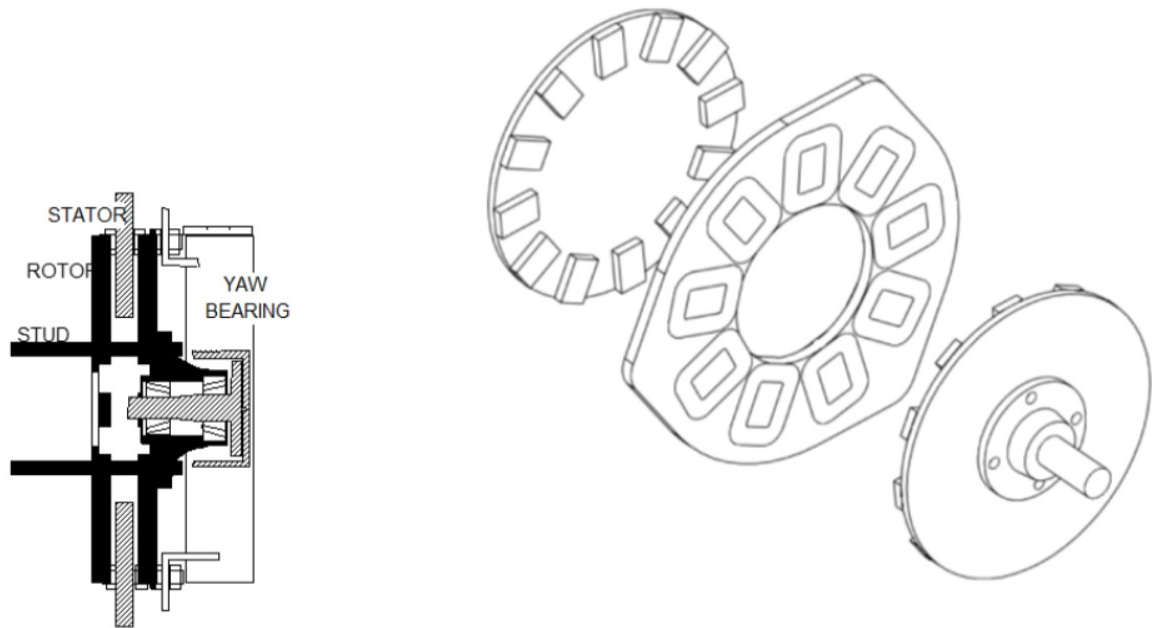
$v$  is the wind speed

$C_p$  is the power coefficient

It is seen that the mechanical power  $P_{mech}$  is proportional to the cube of the wind speed  $v$ .

Students were able to use a spreadsheet to experiment with the equation, applying suitable parameters for their prototype wind turbine design. It is evident that the power coefficient will decrease at high wind speeds due to the increase in the attack angle on the fixed blades, limiting the power output from the turbine. An optimum power output occurs for wind speeds of around 12 metres per second.

Once design work was completed, construction of the mechanical and electrical components could begin. The turbine incorporates an alternator, constructed from a stationary disc of wound electrical coils, and two rings of permanent magnets connected to the rotating blade assembly (Piggott, 2011).



**Figure 117:** The electrical alternator

A final consideration is the storage of the electrical energy produced. This can make use of a lead acid battery. Output from the alternator is converted to direct current by means of a diode circuit, similar to the system described earlier in this chapter. A control circuit is also required which will feed current to the battery when it is not fully charged, but will disconnect the input current when the battery charging is complete.

### Summary

In this chapter we have looked at several very different projects which have involved students in practical numeracy, problem solving, measurement or data collection. Each project has been deeply integrated into the students' main vocational course, and has allowed experimentation with mathematical concepts ranging from the plotting and interpretation of graphs and the application of formulae, to the use of geometry and trigonometry. Students have been highly motivated through producing a product, or by obtaining results which provide important insight into interesting problems.